where the general expression for Q is

$$Q = 1.13 \left[\int_{\theta_w}^{1} \frac{\rho k}{\rho_e k_e} d\theta \int^{1} \frac{c_p^*}{c_{pe}} d\theta^* \right]^{1/2} \qquad \theta \equiv \frac{T - T_0}{T_e - T_0} \tag{4}$$

For Edwards and Tellep's power-law properties

$$k/k_e = \theta^r \qquad \rho/\rho_e = \theta^s \qquad c_p/c_{pe} = \theta^t \tag{5}$$

the expression for Q becomes

$$Q = \frac{1.13}{(t+1)^{1/2}} \left[\frac{1 - \theta_w^{r+s+1}}{r+s+1} - \frac{1 - \theta_w^{r+s+t+2}}{r+s+t+2} \right]^{1/2} \tag{6}$$

Q is related to the Ω' of Ref. 1 by Eq. (3), and Ω is related to θ by

$$\Omega = \theta^{r+s+1} \tag{7}$$

Thus, for cold walls, Eq. (6) is a correlation formula for the curves of Fig. 1 of Ref. 1. The expression for Q has been found accurate to within 3% for $0 < \theta_w < 1$, $\frac{1}{2} < r < \frac{5}{2}$, $s = -1$, $t = 0$ and should be equally accurate for other values of s and t.

Reference 4 also contains expressions for upper and lower bounds on the exact value of Q, involving integrals similar to the ones given in Eq. (4), as well as an analysis for stagnation points at Prandtl numbers around unity.

References

[1] Edwards, D. K. and Tellep, D. M., "Heat transfer in low Prandtl number flows with variable thermal properties," ARS J. **31**, 652–654 (1961).

[2] Jepson, B-M., "Heat transfer in a completely ionized gas," Magnetogasdynamics Lab. Rept. 61-7, Dept. of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. (July 1961).

[3] Fay, J. A. and Kemp, N. H., "Theory of end wall heat transfer in a monatomic gas, including ionization effects," Research Rept. 166, Avco-Everett Research Lab., Everett, Mass. (1963).

[4] Kemp, N. H., "Approximate analytical solution of similarity boundary layer equations with variable fluid properties," Publ. 64-6, Fluid Mechanics Lab., Dept. of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. (1964).

Reply by Authors to N. H. Kemp

D. K. EDWARDS*
University of California, Los Angeles, Calif.
AND
D. M. TELLEP†
Lockheed Aircraft Corp., Sunnyvale, Calif.

IN the preceding note Kemp makes the point that the solution to the momentum equation for inviscid flow is $u = u_e$ only when the fluid has constant density or when the flow is over a flat plate. The point is obvious and needs no laboring. Edwards and Tellep in their 1961 note[1] ruled out other cases with the statement "Only for the flat plate with $du_e/dx = 0$ and $dP/dx = 0$ will the analysis presented hold for fluids with pressure dependent properties." What was intended was to rule out the compressible fluid. The authors simply overlooked ruling out the hypothetical fluid which is incompressible but capable of significant thermal expansion or contraction. It is this hypothetical fluid that is the concern of Kemp, not to treat, but to eliminate from consideration.

An observation on the original note and the present one by Kemp is that the original exact results bear out the use of a reference temperature as suggested by Rubesin and Johnson[2] and Eckert.[3] Derivation of an expression for the reference temperature is facilitated by fitting the exact solution for $\Omega'(0)/(1 - \Omega_w)$ with $(2/\pi^{1/2})[(1 - am) + am\Omega_w]$, where a is adjusted for the best fit.[4] When T_w is not greatly different from T_e, a linearized result gives very nearly the result recommended by Eckert. The reference temperature concept gives results as useful as the approximate relation of Kemp or the previous exact results.

References

[1] Edwards, D. K. and Tellep, D. M., "Heat transfer in low Prandtl number flows with variable thermal properties," ARS J. **31**, 652–654 (1961).

[2] Rubesin, M. W. and Johnson, H. A., "A critical review of skin friction and heat transfer solutions of the laminar boundary layer on a flat plate," Trans. Am. Soc. Mech. Engrs. **71**, 383 (1949).

[3] Eckert, E. R. G., "Engineering relations for heat transfer and friction in high velocity laminar and turbulent boundary layer flow over a surface with constant pressure and temperature," Trans. Am. Soc. Mech. Engrs. **78**, 1273 (1956).

[4] Zwick, E., unpublished solution to homework problem (1962).

Received June 22, 1964.

* Associate Professor, Department of Engineering. Member AIAA.

† Manager, Launch and Entry Thermodynamics, Missile and Space Division. Member AIAA.

Comment on "Method for the Determination of Velocity Distribution in a Thin Liquid Film"

R. M. NEDDERMAN*
University of Cambridge, Cambridge, England

IN a recent note, Persson[1] described a method of measuring velocity profiles in thin films of liquid. This was done by photographing, with a finite exposure, small particles moving with the fluid. The velocities were obtained by measuring the length of the resultant images. The frequency with which various velocities were observed was determined, and hence, by assuming that the particles were uniformly distributed throughout the liquid, the velocity profile could be calculated.

Recent work by Jeffrey[2] has shown that in full tube Poiseuille flow there is a marked tendency for solid particles to migrate across the streamlines and take up a position at about one-third of the radius from the axis. Although this complete migration is admittedly slow, the region near the tube wall rapidly becomes deficient in particles.

There is no reason to assume that this effect does not occur in film flow. If it does, it will produce a particle-deficient layer near the wall, and so velocity profiles determined by Persson's method would have an error in the distance scale. This would be in such a direction as to give a positive velocity at the wall. Inspection of Persson's curves shows that this tendency does in fact exist, and that the unexpected kink in his velocity profiles arises because the profile has been forced to pass through $V = 0$ at $\delta/\delta_0 = 0$.

Some years ago the author[3–5] published an account of a method of measuring velocity profiles in thin films of liquid which avoids the difficulties inherent in Persson's method. Instead of photographing the particles with one camera only, two were used, inclined at an angle of about 40° to each other. The velocities were determined by the same method as Persson used, but the distances from the wall were found from the stereoscopic effect, i.e., the relative positions of the images on the two films. This technique is, in principle, the same as that used for the determination of altitude in aerial surveying. It was found possible by this technique to measure distances across the film to an accuracy of about ±0.001 cm,

Received May 25, 1964.

* Praelector and Fellow of Darwin College, University Demonstrator in Chemical Engineering.

though there is no fundamental reason why this should not be improved.

Reference

[1] Persson, S. L., "Method for determination of velocity distribution in a thin film," AIAA J. **2**, 372–373 (1964).

[2] Jeffrey, R. C., "Particle motion in Poiseuille flow," Ph.D. Thesis, Univ. of Cambridge, (1964).

[3] Nedderman, R. M., "Velocity profiles in thin liquid layers," Ph.D. Thesis, Univ. of Cambridge (1960); available in microfilm from Micro Methods Ltd., East Ardsley, Wakefield, Yorks, England.

[4] Nedderman, R. M., "The use of stereoscopic photography for the measurement of velocities in liquids," Chem. Eng. Sci. **16**, 113 (1961).

[5] Wilkes, J. O. and Nedderman, R. M., "The measurement of velocities in thin films of liquid," Chem. Eng. Sci. **17**, 177 (1962).

Reply by Author to R. M. Nedderman

S. Lennart Persson*

Flygmotor Aeroengine Company, Trollhättan, Sweden

Nedderman observes that the main assumption in my method concerning the uniform distribution may be doubtful. This argument was based on experiments done by Jeffrey in full tube Poiseuille flow. According to the comment, Jeffrey's experiments have shown that migration may occur. The physical reasons for this migration are, however, not clear to the author from the comment, and it is thus difficult to imagine what kind of effect one would expect in thin water films.

From the velocity profiles given in Ref. 1 it may be noticed that $dv/d\delta$ approaches zero at $\delta = 0$. This may be regarded as supporting Nedderman's comment. In profiles measured at lower mean film velocities (and thus thicker films), this effect does not occur, however, as can be observed from Fig. 1.

Another possible explanation for the slope of the profiles given in Ref. 1 might be slip flow due to the use of particles that are too big compared to the film thickness. The results given in Fig. 1 were obtained with particles of the same size but with films of the order of five times the thickness of the previously studied rapid films. However, this eventual slip flow effect needs further investigation.

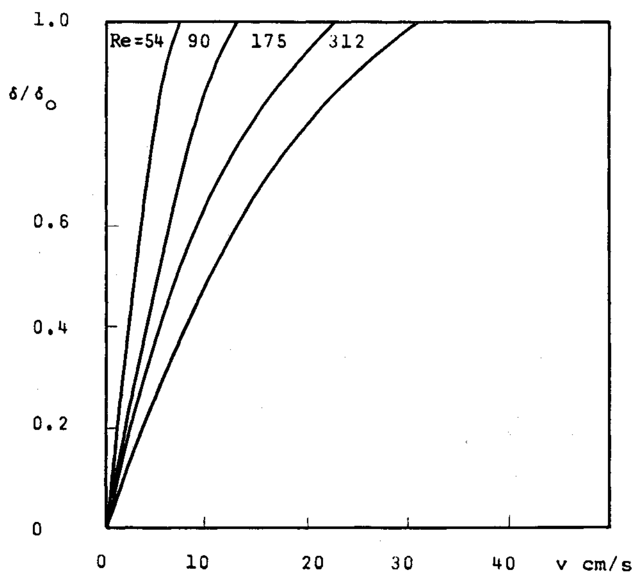

Fig. 1 Influence of Reynolds number on velocity profiles without concurrent air flow.

Received June 29, 1964.

* Head, Physics Section, Research Department.

Reference

[1] Persson, S. L., "Method for determination of velocity distribution in a thin liquid film," AIAA J. **2**, 372–373 (1964).

Erratum: "Comment on 'Equilibrium Orientations of Gravity-Gradient Satellites'"

Thomas R. Kane* and Peter W. Likins†

Stanford University, Stanford, Calif.

[AIAA J. **2**, 1357 (1964)]

In the above Technical Comment, Eq. (3) should read

$$C\ddot{\gamma} + \Omega^2(A - B)\gamma - \Omega(A - B - C)\dot{\beta} = 0$$

The symbol $\ddot{\gamma}$ dropped out after final page proofs had been released.

Recieved July 27, 1964.

* Professor of Engineering Mechanics. Member AIAA.

† Graduate Student.

One-Dimensional Rayleigh Flow of a Partially Ionized Gas

Malcolm McChesney*

The University of Liverpool, Liverpool, England

Nomenclature

ρ	= gas density nm, where n is the total number of heavy particles per cubic centimeter, i.e., the number density of ions and neutral atoms (not the number density of neutral atoms alone as suggested by Yen)
A	= area of the channel
u	= gas velocity in the channel
q	= energy supplied per cubic centimeter per second to the flowing gas
m	= mass of a neutral atom
m_e	= mass of an electron
α	= degree of ionization
T	= absolute static temperature of the flowing gas
X	= ionization potential of the neutral atom
p	= static pressure of the flowing gas
$2\pi\hbar$	= Planck's constant
h	= specific enthalpy of the flowing gas
$(g_0)_a, (g_0)_i$	= ground state statistical weights of the neutral atom and first ion, respectively

In a recent note Yen[1] has studied the Rayleigh flow of a partially ionized gas and concluded that "choking at Mach number equal to 1" cannot be obtained. The purpose of the present note is to demonstrate the incorrectness of the basic equations, and the conclusion, of Yen.

Received January 23, 1964. This work is financed by the Department of Scientific and Industrial Research, through a contract from the Aero Division of the National Physical Laboratory, England.

* Lecturer-in-Charge, Shock Tube Group.